## Area and Perimeter

This problem gives you the chance to:

- work with area and perimeter of rectangles

1. The perimeter of this rectangle is $2(5+2)=14$ inches.

The area of this rectangle is $2 \times 5=10$ square inches.


5 inches
a. Draw a diagram of a rectangle with the same perimeter, but a larger area. Write down the area of your rectangle.
b. Draw a diagram of a rectangle with the same perimeter, but a smaller area.

Write down the area of your rectangle.
2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter?
Explain your reasoning.

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter?

Explain your reasoning.
$\qquad$
$\qquad$
$\qquad$

| Area and Perimeter | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: - work with area and perimeter of rectangles <br> Based on these, credit for specific aspects of performance should be assigned as follows | $\underset{\mathrm{s}}{\mathrm{point}}$ | $\begin{gathered} \text { sectio } \\ n \\ \text { points } \end{gathered}$ |
| 1.a Draws a rectangle with sides such as: 3 inches x 4 inches area $=12$ square inches <br> Draws a rectangle with sides such as: 1 inch $\times 6$ inches area $=6$ square inches <br> b. | $1$ | 4 |
| 2.a Gives correct answer: Yes and <br> Gives correct explanation such as: <br> Area $2 \times 12=24$, Perimeter $2(2+12)=28$ inches or <br> Area $1 \times 24=24$, Perimeter $2(1+24)=50$ inches <br> Partial credit <br> Allow partial credit for a partially correct answer. <br> Gives correct answer: Yes and <br> Gives correct explanation such as: <br> b. Area $4 \times 6=24$, Perimeter $2(4+6)=20$ inches <br> Partial credit <br> Allow partial credit for a partially correct answer. | 3 <br> or <br> 3 <br> (2) <br> 3 <br> (2) | 6 |
| Total Points |  | 10 |

## Percents

Work the task and look at the rubric. What does a student need to understand to be successful on this task?

Look at student work for part 1. How many students gave answers, such as:

- A complete response with rectangles of correct dimensions that fit the constraints of the same perimeter and a larger or smaller perimeter and quantifified the area for the new shape?
- Gave rectangles with correct dimensions but didn't give a value for area?
- Gave rectangles that had larger or smaller areas, but also changed the perimeter?
- Used the same rectangle of $2 \times 5$ ?
- Gave shapes that were not rectangles?

What misconceptions did you see in student work?
Now look at the work for part 2 a and 2 b . How many students gave answers, such as:

- Used correct dimensions, showed that area is the same, and calculated the new perimeters?
- Used correct dimensions but didn't quantify why or how they fit the constraints?
- Thought one was possible and the other was not possible?
- Thought changing the area would change the dimensions?

What other types of errors did you see in students thinking?
How do you communicate values around justifying an answer in your classroom? Do students know that quantifying is an important mathematical value?
How often do students have the opportunity to do investigations in your classroom? What misunderstandings or habits of mind prevented students from making attempts to find solutions in part 2?

Although incomplete, what pieces of thinking showed that students had some understanding of the concepts? What would need to be done to these answers to get a complete solution? How could you use this idea to develop a class discussion?

## Looking at Student Work on Area and Perimeter

Student A is able to find rectangles to meet the constraints. The student is able to put numbers into a formula to calculate perimeter and also calculates the area. Notice that the student uses greater than or less than to compare new calculations to originals to complete the proof.

## Student A


b. Draw a diagram of a rectangle with the same perimeter, but a smaller area $\qquad$
Write down the area of your rectangle. $\quad$ perimeter $=2 \times(6+1)=14$ inches


Ginches
2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter?
Explain your reasoning.
$\checkmark$
Yes because 12 and 2 multiplied equals
8 inches

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning.


Student B is able to meet the demands of the task for part 1. In part 2 the student shows an understanding of how to change the shape and restrictions about what types of numbers can be used for the dimensions, but doesn't use specific examples to complete the argument that it is possible to change perimeter without changing the area.

## Student B

2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? Explain your reasoning.

the 8 inches into a larger number, and the 3inches:

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter?

Explain your reasoning.


Student C is able to find rectangles to justify part 2 . However the student does not notice the constraint of keeping the perimeter the same, when solving part 1. How do we help students develop the habit of mind to look for all the constraints and check that they have been met?
Student C

1. The perimeter of this rectangle is $2(5+2)=14$ inches. 5
The area of this rectangle is $2 \times 5=10$ square inches.

a. Draw a diagram of a rectangle with the same perimeter, but a larger area.

b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle.

$$
4 \text { Ain } 1 \text { tined }=45 \text { in }
$$

2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter?
Explain your reasoning.
Yes, $12 \cdot 2=24 \quad 12+12=24+$


Student D is able to find correct dimensions for rectangles to meet the criteria for part 1 . In part 2 the student uses different logic for part a and $b$. In part a the student has the common misconception that a larger perimeter must have a larger area. In part $b$ the student knows that the dimensions need to be factors of 24, but only tests one case which doesn't work. What might be next steps for this student?

## Student D

a. Draw a diagram of a rectangle with the same perimeter, but a larger area. 5 inches Write down the area of your rectangle.

b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your/rectangle.

2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter?

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning.


Student E makes nonrectangular shapes in part 1, but still uses the formula for area of a rectangle. How do students develop the connection between shape and possible dimensions? What types of experiences does this student need? In part 2 the student can do part ib, but doesn't think the dimensions can be changed in aa. Why do you think students have these kinds of inconsistencies?
Student E
The area of this rectangle is $2 \times 5=10$ square inches.

a. Draw a diagram of a rectangle with the same perimeter, but a larger area.

Write down the area of your rectangle.

b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle.

$0 i$
2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter?
Explain your reasoning.

$\qquad$
b. Is it possible to draw a rectangle with the same area, but a smaller perimeter?

Explain your reasoning.

$\qquad$

## Area and Perimeter

| Student Task | Work with area and perimeter of rectangles. Investigate how change <br> linear measures effects area and perimeter. |
| :--- | :--- |
| Core Idea 4 | Analyze characteristics and properties of two-dimensional <br> geometric shapes. Apply appropriate techniques, tools, and formulas to <br> Geometry <br> and |
| Measurement | - $\quad$Develop, understand, and use formulas to determine area of <br> quadrilaterals. <br>  <br>  <br> Investigate, describe, and reason about the results of subdividing, <br> combining, and transforming shapes. |

The mathematics of this task:

- Willingness to investigate different cases to see if something is possible
- Identifying constraints
- See relationships between dimensions and the measurements of area and perimeter
- Quantify measurements in order to make comparisons
- Understanding which dimensional measurements will and will not make a rectangle and why
- Understanding that rectangles have two sets of parallel sides with the same side length for each set
- Calculating area and perimeter
- Understanding the logic of justification
- Understanding and tracking units

Based on teacher observations, this is what sixth graders knew and were able to do:

- Rectangles have four sides
- Know formulas for area and perimeter and how to use them to calculate
- Draw rectangles and give appropriate dimensions


## Areas of difficulty for sixth graders:

- Confusing perimeter with diameter, so they kept one dimension the same and changed to height to get the new areas
- Conducting an investigation. Some students did not believe it was possible to keep the perimeter the same and change the area, but they did not try numbers to test their conjectures.
- Applying knowledge to a complex problem
- Understanding that opposite sides of a rectangle needed to be the same size
- Making connections between area and factors to develop possible side lengths
- Using all the constraints in the problem
- Changing the wrong measurement
- Thought more square-like shapes were no longer rectangles
- Thought that making changes in one of the dimensions would change both measurements
Strategy used by successful students:
- Thinking about factors to determine possible dimensions for the rectangle


## Task 5 - Area \& Perimeter

Mean: 3.11 StdDev: 3.77

Table 34: Frequency Distribution of MARS Test Task 5, Grade 6

| Task 5 <br> Scores | Student <br> Count | \% at or <br> below | $\%$ at or <br> above |
| :---: | :---: | :---: | :---: |
| 0 | 3088 | $48.6 \%$ | $100.0 \%$ |
| 1 | 203 | $51.8 \%$ | $51.4 \%$ |
| 2 | 465 | $59.1 \%$ | $48.2 \%$ |
| 3 | 123 | $61.0 \%$ | $40.9 \%$ |
| 4 | 594 | $70.3 \%$ | $39.0 \%$ |
| 5 | 103 | $72.0 \%$ | $29.7 \%$ |
| 6 | 215 | $75.3 \%$ | $28.0 \%$ |
| 7 | 333 | $80.6 \%$ | $24.7 \%$ |
| 8 | 227 | $84.1 \%$ | $19.4 \%$ |
| 9 | 144 | $86.4 \%$ | $15.9 \%$ |
| 10 | 864 | $100.0 \%$ | $13.6 \%$ |

Figure 43: Bar Graph of MARS Test Task 5 Raw Scores, Grade 6


The maximum score available for this task is 10 points.
The minimum score needed for a level 3 response, meeting standards, is 5 points.
About half the students could give the dimensions for rectangles with the same perimeter and larger or smaller areas. Some students, about $39 \%$ could also calculate the area of the new rectangles. A few students, $30 \%$, could give the dimensions for rectangles with the same perimeter and larger and smaller areas, calculate the areas for the new rectangles, and give the dimensions for a rectangle with the same area and either a larger or smaller perimeter. Almost $14 \%$ of the students could meet all the demands of the task including giving the dimensions for a rectangle with the same area and larger and smaller perimeters and justify the dimensions by giving the new perimeters. Almost half the students scored no points on this task. $81 \%$ of the students with this score attempted the task.

## Area and Perimeter

| Points | Understandings | Misunderstandings |
| :---: | :--- | :--- |
| $\mathbf{0}$ | $81 \%$ of the students with this <br> score attempted the task. | Students could not give dimensions for <br> rectangles with the same perimeter. <br> Students gave dimensions for shapes that <br> were not rectangular. Students gave shapes <br> with dimensions that did not add to 14. |
| $\mathbf{2}$ | Students could give dimensions <br> for rectangles with the same <br> perimeter but larger and smaller <br> areas. | Students did not justify their solution by <br> calculating the new areas. |
| $\mathbf{4}$ | Students could give dimensions <br> for rectangles with the same <br> perimeter but larger and smaller <br> areas and justify their solution <br> by calculating the new areas. | Students did not understand how to change <br> the perimeter while maintaining the same <br> area. 14\% said that if the area is larger, <br> then the perimeter must be larger. Another <br> 8\% said that if the area stays the same, the <br> perimeter will stay the same. . 0\% thought <br> it was possible, but did not attempt to give <br> the new dimensions or test their <br> conjectures. |
| $\mathbf{5}$ | Students could give dimensions <br> of new rectangles, for parts 1 <br> and 2, but in general did not <br> calculate the area or perimeter <br> for the new shapes. | Students don't understand the logic of <br> justification. Once they have a dimension <br> they are done with the task. |
| $\mathbf{8}$ |  | Students did not calculate the perimeters in <br> part 2 of the task. |
| $\mathbf{1 0}$ | Students could investigate <br> hypotheses, such as maintaining <br> perimeter and changing the <br> area, and then calculate the <br> values for area and perimeter of <br> the new shape to verify the <br> hypothesis. |  |

## Implications for Instruction

Students at this grade level have been working with area and perimeter since third grade. They know the basic procedures for making the calculations on rectangles. However, many students are still struggling with the conceptual knowledge. They haven't learned the information in a way that allows them to apply it to new or unusual situations.

Consider the idea of layers of knowledge. One way of knowing something is physical representation (Using only the grid paper provided, construct as many rectangles as you can with an area of 12 square centimetres.) The next level of understanding is to be able to apply an idea to a real world context. (Fred's flat has five rooms. The total floor area is 60 square meters. Draw a plan of Fred's flat. Label each room, and show the dimensions of all the rooms.) Finally, students should be able to work with mathematical abstractions. (The area of a rectangle is 12 square centimetres. What might be the dimensions?) This idea of assessment by contextual exhaustion is from the work of David Clarke, University of Melbourne.

As students at this grade level have been working with the calculations and procedures at earlier grade levels, the challenge at this grade level is to work more on cognitive demand by applying the information in new and novel ways, increasing the level of abstraction, and providing problems with longer reasoning chains. Students need opportunities to investigate the relationships between mathematical ideas, such as area and perimeter. They should have tools for organizing their thinking. Students need to develop persistence for solving problems where the correct procedure is not immediately obvious; in this case working from the answer to the dimensions. Students should also be developing criteria for making a convincing argument, by quantifying information and checking it against the original conjectures.

## Ideas for Action Research - Examining Student Misconceptions and Planning a Re-engagement Lesson

Almost $49 \%$ of the sixth graders scored " 0 " on this task. $80 \%$ of the students with this score attempted the task, amounting to almost 3000 students. For this reason, it is important to look at their thinking in depth. Students have been working with area and perimeter of rectangles since third grade. What made this difficult for students? There are some important misconceptions. Errors are more than just an unwillingness to investigate. The following student work is from students with total scores of 14 to 25 .

Questions for consideration when examining student work:

- Are students making the areas larger and smaller in part 1 ?
- Are students forgetting to keep the perimeter the same?
- Are students giving dimensions that will make a rectangle?
- What does in mean to have a dimension of " 0 "?
- Is the student changing the wrong measurement, area or perimeter?
- Is the thinking consistent in both parts of 2, thinking they are both possible or both impossible? What are some of the reasons for the discrepancies?
- Some students describe shapes that would meet the new criteria for either 2 a or 2 b , but don't give dimensions or offer proof by verifying area and perimeter. Can you find these?
- Several students use the same dimensions in part one as the sample rectangle. Why might they do this? What aren't they grasping about the prompt?
- Some of the student reasoning is based on beliefs about the effects of operations on numbers. What is correct and incorrect in their understanding?

After examining and discussing all the student work, how might you plan a small mini-unit to dig into the misconceptions and help students explore area and perimeter to develop a deeper level of understanding? Which misconceptions should be dealt with explicitly? Which misconceptions should be dealt with through investigation and direct experience?

## Student 1

1. The perimeter of this rectangle is $2(5+2)=14$ inches.

The area of this rectangle is $2 \times 5=10$ square inches.

a. Draw a diagram of a rectangle with the same perimeter, but a larger area. Write down the area of your rectangle.

b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle.

2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? Explain your reasoning.

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter?


## Student 2

a. Draw a diagram of a rectangle with the same perimeter, but a larger area 5 inches Write down the area of your rectangle.

$$
2(0+3)=10 \text { inches }
$$

b. Draw a diagram of a rectangle with the same perimeter, but a smaller area.

Write down the area of your rectangle.

2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as
the one on the right, but a larger perimeter?

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter?


## Student 3

a. Draw a diagram of a rectangle with the same perimeter, but a larger area.

Write down the area of your rectangle.
b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle.

5 inches

2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter?


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## Student 4


2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter?
$\int$ Explain your reasoning.

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter?

Explain your reasoning.


## Student 5

a. Draw a diagram of a rectangle with the same perimeter, but a larger area. 5 inches Write down the area of your rectangle.

$$
p=3+(6+3)=1 \quad \mathrm{Q}=3 \times 6=18
$$


b. Draw a diagram of a rectangle with the same perimeter, but a smaller area.

Write down the area of your rectangle.

$$
\begin{aligned}
& 2(4+2) \\
& 2 \cdot 6=12
\end{aligned} a x
$$

2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? Explain your reasoning.

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter?


Grade 6 - 2008

## Student 6

a. Draw a diagram of a rectangle with the same perimeter, put a larger area.

Write down the area of your rectangle.

2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as
the one on the right, but a larger perimeter?
Explain your reasoning.

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter?

Explain your reasoning.

rectangle with the same area


$$
\text { and } s \text { maller perimeter. } 1 \text { a }
$$

## Student 7

a. Draw a diagram of a rectangle with the same perimeter, but a larger area. 5 inches

Write down the area of your rectangle.

b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle.

2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? Explain your reasoning.

not matter about the leghth or

$$
\text { height. X } X
$$

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning.


Grade 6 - 2008

## Student 8

Can you figure out what the student is calculating in part 1. In $2 b$ the student gives the perimeter for the correct rectangle, but no dimensions. What do you think the student understands?

1. The perimeter of this rectangle is $2(5+2)=14$ inches.

The area of this rectangle is $2 \times 5=10$ square inches.
$\times$
a. Draw a diagram of a rectangle with the same perimeter, but a larger area.

5 inches Write down the area of your rectangle. .

b. Draw a diagram of a rectangle with the same perimeter, but a smaller area.

Write down the area of your rectangle.

2. The perimeter of this rectangle is 82 inches. The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? Explain your reasoning.
$\qquad$
b. Is it possible to draw a rectangle with the same area, but a smaller perimeter?

Explain your reasoning.


## Student 9

a. Draw a diagram of a rectangle with the same perimeter, but a larger area.

5 inches
Write down the area of your rectangle.

b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle.

2. The perimeter of this rectangle is 22 inches.

The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as
the one on the right, but a larger perimeter?
Explain your reasoning.

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter?

Explain your reasoning.


## Student 10



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## Student 11

write down the area of your rectangle.

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning.
 the outside and the area is the o inside. Different things. X

## Student 12

a. Draw a diagram of a rectangle with the same perimeter, but a larger area.

5 inches
Write down the area of your rectangle.

b. Draw a diagram of a rectangle with the same perimeter, but a smaller area.

Write down the area of your rectangle. 3

2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches
a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter?
Explain your reasoning.

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter?


Grade 6 - 2008

## Ideas for Action Research 2 - Problem of the Month

One interesting task to help students stretch their thinking about 3-dimensional shapes is the problem of the month: Surrounded and Covered, from the Noyce Website:
www.noycefdn.org/math/members/POM/pom.html
Ask students to work individually or in teams to solve the problem. Have them make posters of calculations they made, their conclusions, and graphics or visuals to support their thinking. The poster might also include other ideas they want to explore or conjectures they haven't had time to test. The purpose is to give them some complex mathematical thinking, that requires persistence, willingness to make mistakes, edit and revise, and is worth understanding the thinking of others. Now give students a chance to investigate part D of this task.

## Level D:

Tex has a home on the range (where the deer and antelope play). He wants to build a rectangular corral for his horses. He only has 170 feet of fencing. What size of corral should be built to make sure the horses have the most room? List the dimensions and area. Justify why you know you know the corral is as large as possible. Explain how you figured it out.

Tex gets a better idea. He has a barn on his ranch that is 80 feet by 120 feet. He decides to build the corral using one side of the barn and the 170 feet of fencing. What should be the dimensions of the corral now? What is the area? Explain your mathematical reasoning.

A year later, Tex needs a second corral. This time he has 240 feet of fencing. He doesn't want to use the other side of the barn, because it is near a small pond. He picks out a new location and realizes that he does not need to make the corral a rectangle. He designs a corral the shape of a hexagon. What are the lengths of the sides and what is the area of the corral? Explain how you found your answer.

Tex thinks that maybe another shape would make an even larger area for his corral. Determine what the shape should be and its area and dimensions. Justify your answer using mathematical reasoning.

