Area and Perimeter

This problem gives you the chance to:work with area and perimeter of rectangles

Write down the area of your rectangle.

1. The perimeter of this rectangle is 2(5 + 2) = 14 inches.
2 inches
2 inches
a. Draw a diagram of a rectangle with the same perimeter, but a larger area.

- b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle.
- 2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches
 a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? Explain your reasoning.

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning.

10

Are	Rubric		
The core elements of performance required by this task are: • work with area and perimeter of rectangles			
Based on these, credit for specific aspects of performance should be assigned as follows			sectio n points
1.a	Draws a rectangle with sides such as: 3 inches x 4 inches	1	
•	area = 12 square inches	1	
	Draws a rectangle with sides such as: 1 inch x 6 inches	1	
	area = 6 square inches	1	
b.			4
2.a	Gives correct answer: Yes and		
•	Gives correct explanation such as: Area $2 \times 12 = 24$. Parimeter $2(2 \pm 12) = 28$ inches	3	
	Area 2 x $12 - 24$, refinitelet $2(2 + 12) - 28$ inches or	or	
	Area 1 x $24 = 24$, Perimeter $2(1 + 24) = 50$ inches	3	
	Partial credit		
	Allow partial credit for a partially correct answer.	(2)	
	Gives correct answer: Yes and Gives correct explanation such as:		
b.	Area 4 x $6 = 24$, Perimeter $2(4 + 6) = 20$ inches	3	
	Partial credit	(2)	
	Allow partial credit for a partially correct answer.	(2)	6
	Total Points		10

Percents

Work the task and look at the rubric. What does a student need to understand to be successful on this task?

Look at student work for part 1. How many students gave answers, such as:

- A complete response with rectangles of correct dimensions that fit the constraints of the same perimeter and a larger or smaller perimeter and quantifified the area for the new shape?
- Gave rectangles with correct dimensions but didn't give a value for area?
- Gave rectangles that had larger or smaller areas, but also changed the perimeter?
- Used the same rectangle of 2 x 5?
- Gave shapes that were not rectangles?

What misconceptions did you see in student work?

Now look at the work for part 2a and 2b. How many students gave answers, such as:

- Used correct dimensions, showed that area is the same, and calculated the new perimeters?
- Used correct dimensions but didn't quantify why or how they fit the constraints?
- Thought one was possible and the other was not possible?
- Thought changing the area would change the dimensions?

What other types of errors did you see in students thinking?

How do you communicate values around justifying an answer in your classroom? Do students know that quantifying is an important mathematical value?

How often do students have the opportunity to do investigations in your classroom? What misunderstandings or habits of mind prevented students from making attempts to find solutions in part 2?

Although incomplete, what pieces of thinking showed that students had some understanding of the concepts? What would need to be done to these answers to get a complete solution? How could you use this idea to develop a class discussion?

Looking at Student Work on Area and Perimeter

Student A is able to find rectangles to meet the constraints. The student is able to put numbers into a formula to calculate perimeter and also calculates the area. Notice that the student uses greater than or less than to compare new calculations to originals to complete the proof.

Student A



Student B is able to meet the demands of the task for part 1. In part 2 the student shows an understanding of how to change the shape and restrictions about what types of numbers can be used for the dimensions, but doesn't use specific examples to complete the argument that it is possible to change perimeter without changing the area.

Student B

2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inchesa. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? Explain your reasoning.	3 inches
les, it is possible. If you change 8 inches	
the 8 inches into a larger number and the Binches	inch.
to a smaller number, you get) a larger perimeter, but	(2)
both numbers need to be factors of 24. (2	<u></u>
b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning.	1 -
Yes, it is possible. If you change the Sinches into a	(2)
smaller number, and the Binches linto, a larger num	iber 12
you get a smaller perimeter, but both numbers need to	ibe
incrois of and. (8)	10

Student C is able to find rectangles to justify part 2. However the student does not notice the constraint of keeping the perimeter the same, when solving part 1. *How do we help students develop the habit of mind to look for all the constraints and check that they have been met?*

Student C



Student D is able to find correct dimensions for rectangles to meet the criteria for part 1. In part 2 the student uses different logic for part a and b. In part a the student has the common misconception that a larger perimeter must have a larger area. In part b the student knows that the dimensions need to be factors of 24, but only tests one case which doesn't work. *What might be next steps for this student?*

Student D

5 inches a. Draw a diagram of a rectangle with the same perimeter, but a larger area. Write down the area of your rectangle. b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle. 6 2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? 3 inches Explain your reasoning. 8 inches

b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning.

Student E makes nonrectangular shapes in part 1, but still uses the formula for area of a rectangle. *How do students develop the connection between shape and possible dimensions? What types of experiences does this student need?* In part 2 the student can do part 2b, but doesn't think the dimensions can be changed in 2a. *Why do you think students have these kinds of inconsistencies?*

Student E 2 inches The area of this rectangle is $2 \times 5 = 10$ square inches. 5 inches a. Draw a diagram of a rectangle with the same perimeter, but a larger area. Write down the area of your rectangle. = 4+4+2=14 00 b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle. OC - 10+2+1+1=14 00 =.16 X 2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? 3 inches Explain your reasoning. 8 inches b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning. aus Gx11=211 and 12+8=20VV

6th Grade

Student Task	Work with area and perimeter of rectangles. Investigate how change		
	linear measures effects area and perimeter.		
Core Idea 4	Analyze characteristics and properties of two-dimensional		
Geometry	geometric shapes. Apply appropriate techniques, tools, and formulas to		
and	determine measurements.		
Measurement	• Develop, understand, and use formulas to determine area of		
	quadrilaterals.		
	• Investigate, describe, and reason about the results of subdividing,		
	combining, and transforming shapes.		

The mathematics of this task:

- Willingness to investigate different cases to see if something is possible
- Identifying constraints
- See relationships between dimensions and the measurements of area and perimeter
- Quantify measurements in order to make comparisons
- Understanding which dimensional measurements will and will not make a rectangle and why
- Understanding that rectangles have two sets of parallel sides with the same side length for each set
- Calculating area and perimeter
- Understanding the logic of justification
- Understanding and tracking units

Based on teacher observations, this is what sixth graders knew and were able to do:

- Rectangles have four sides
- Know formulas for area and perimeter and how to use them to calculate
- Draw rectangles and give appropriate dimensions

Areas of difficulty for sixth graders:

- Confusing perimeter with diameter, so they kept one dimension the same and changed to height to get the new areas
- Conducting an investigation. Some students did not believe it was possible to keep the perimeter the same and change the area, but they did not try numbers to test their conjectures.
- Applying knowledge to a complex problem
- Understanding that opposite sides of a rectangle needed to be the same size
- Making connections between area and factors to develop possible side lengths
- Using all the constraints in the problem
- Changing the wrong measurement
- Thought more square-like shapes were no longer rectangles
- Thought that making changes in one of the dimensions would change both measurements

Strategy used by successful students:

• Thinking about factors to determine possible dimensions for the rectangle

Task 5 - Area & Perimeter

Mean: 3.11 StdDev: 3.77

Task 5	Student	% at or	% at or
Secrec	Count	bolow	
Scores	Count	Delow	apove
0	3088	48.6%	100.0%
1	203	51.8%	51.4%
2	465	59.1%	48.2%
3	123	61.0%	40.9%
4	594	70.3%	39.0%
5	103	72.0%	29.7%
6	215	75.3%	28.0%
7	333	80.6%	24.7%
8	227	84.1%	19.4%
9	144	86.4%	15.9%
10	864	100.0%	13.6%

Table 34: Frequency Distribution of MARS Test Task 5, Grade 6

Figure 43: Bar Graph of MARS Test Task 5 Raw Scores, Grade 6



MARS Task 5 Raw Scores The maximum score available for this task is 10 points. The minimum score needed for a level 3 response, meeting standards, is 5 points.

About half the students could give the dimensions for rectangles with the same perimeter and larger or smaller areas. Some students, about 39% could also calculate the area of the new rectangles. A few students, 30%, could give the dimensions for rectangles with the same perimeter and larger and smaller areas, calculate the areas for the new rectangles, and give the dimensions for a rectangle with the same area and either a larger or smaller perimeter. Almost 14% of the students could meet all the demands of the task including giving the dimensions for a rectangle with the same area and larger and smaller perimeters and justify the dimensions by giving the new perimeters. Almost half the students scored no points on this task. 81% of the students with this score attempted the task.

Area and Perimeter

Points	Understandings	Misunderstandings
0	81% of the students with this	Students could not give dimensions for
	score attempted the task.	rectangles with the same perimeter.
		Students gave dimensions for shapes that
		were not rectangular. Students gave shapes
		with dimensions that did not add to 14.
2	Students could give dimensions	Students did not justify their solution by
	for rectangles with the same	calculating the new areas.
	perimeter but larger and smaller	
	areas.	
4	Students could give dimensions	Students did not understand how to change
	for rectangles with the same	the perimeter while maintaining the same
	perimeter but larger and smaller	area. 14% said that if the area is larger,
	areas and justify their solution	then the perimeter must be larger. Another
	by calculating the new areas.	8% said that if the area stays the same, the
		perimeter will stay the same. 10% thought
		the new dimensions or test their
		conjectures
5	Students could give dimensions	Students don't understand the logic of
3	of new rectangles for parts 1	iustification Once they have a dimension
	and 2 but in general did not	they are done with the task.
	calculate the area or perimeter	
	for the new shapes.	
8		Students did not calculate the perimeters in
_		part 2 of the task.
10	Students could investigate	
	hypotheses, such as maintaining	
	perimeter and changing the	
	area, and then calculate the	
	values for area and perimeter of	
	the new shape to verify the	
	hypothesis.	

Implications for Instruction

Students at this grade level have been working with area and perimeter since third grade. They know the basic procedures for making the calculations on rectangles. However, many students are still struggling with the conceptual knowledge. They haven't learned the information in a way that allows them to apply it to new or unusual situations.

Consider the idea of layers of knowledge. One way of knowing something is physical representation (Using only the grid paper provided, construct as many rectangles as you can with an area of 12 square centimetres.) The next level of understanding is to be able to apply an idea to a real world context. (Fred's flat has five rooms. The total floor area is 60 square meters. Draw a plan of Fred's flat. Label each room, and show the dimensions of all the rooms.) Finally, students should be able to work with mathematical abstractions. (The area of a rectangle is 12 square centimetres. What might be the dimensions?) *This idea of assessment by contextual exhaustion is from the work of David Clarke, University of Melbourne*.

As students at this grade level have been working with the calculations and procedures at earlier grade levels, the challenge at this grade level is to work more on cognitive demand by applying the information in new and novel ways, increasing the level of abstraction, and providing problems with longer reasoning chains. Students need opportunities to investigate the relationships between mathematical ideas, such as area and perimeter. They should have tools for organizing their thinking. Students need to develop persistence for solving problems where the correct procedure is not immediately obvious; in this case working from the answer to the dimensions. Students should also be developing criteria for making a convincing argument, by quantifying information and checking it against the original conjectures.

Ideas for Action Research – Examining Student Misconceptions and Planning a Re-engagement Lesson

Almost 49% of the sixth graders scored "0" on this task. 80% of the students with this score attempted the task, amounting to almost 3000 students. For this reason, it is important to look at their thinking in depth. Students have been working with area and perimeter of rectangles since third grade. What made this difficult for students? There are some important misconceptions. Errors are more than just an unwillingness to investigate. The following student work is from students with total scores of 14 to 25.

Questions for consideration when examining student work:

- Are students making the areas larger and smaller in part 1?
- Are students forgetting to keep the perimeter the same?
- Are students giving dimensions that will make a rectangle?
- What does in mean to have a dimension of "0"?
- Is the student changing the wrong measurement, area or perimeter?
- Is the thinking consistent in both parts of 2, thinking they are both possible or both impossible? What are some of the reasons for the discrepancies?
- Some students describe shapes that would meet the new criteria for either 2a or 2b, but don't give dimensions or offer proof by verifying area and perimeter. Can you find these?

- Several students use the same dimensions in part one as the sample rectangle. Why might they do this? What aren't they grasping about the prompt?
- Some of the student reasoning is based on beliefs about the effects of operations on numbers. What is correct and incorrect in their understanding?

After examining and discussing all the student work, how might you plan a small mini-unit to dig into the misconceptions and help students explore area and perimeter to develop a deeper level of understanding? Which misconceptions should be dealt with explicitly? Which misconceptions should be dealt with through investigation and direct experience?

1. The perimeter of this rectangle is $2(5+2) = 14$ inches.	2 inches
The area of this rectangle is $2 \ge 5 = 10$ square inches.	
a. Draw a diagram of a rectangle with the same perimeter, but a larger area. 5 inches Write down the area of your rectangle.	D (D (
b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle.	
2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? Explain your reasoning.	3 inches C ume C
b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning. <u>Het hecause with the your nomber to the same area, but a smaller perimeter?</u> <u>Can take it and make it bigger and the to</u>	000 100 100 100 100
Same answer! A Sixth Grade - Area and	10 Perimeter

5 inches a. Draw a diagram of a rectangle with the same perimeter, but a larger area. Write down the area of your rectangle. 0 0 2(Bt 3 Inches 310204 6 X b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle. 0 ·X =10 ind 6 21 4+1 linch V 2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? 3 inches Explain your reasoning. 0 8 inches b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning. OU more NGC Student 3 5 inches a. Draw a diagram of a rectangle with the same perimeter, but a larger area. Write down the area of your rectangle. 4,0 0 3 b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. 0 Write down the area of your rectangle. 0 Oin 0 0 × 7in 2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches a. Is it possible to draw a rectangle with the same area as 3 inches the one on the right, but a larger perimeter? Explain your reasoning. 0 8 inches 9 b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? \mathbb{C} Explain your reasoning. CA Smo P 0 an 0

Grade 6 – 2008 Copyright © 2008 by Noyce Foundation All rights reserved.

0

2 2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? 3 inch Explain your reasoning. 1 8 inches b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning. W Student 5 5 inches a. Draw a diagram of a rectangle with the same perimeter, but a larger area. Write down the area of your rectangle. pe: 18 D B b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle. 0 -4=9 0 2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? 3 inches Explain your reasoning. 8 inches 0 ma lines out 60 aga b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning. ß

Grade 6 – 2008 Copyright © 2008 by Noyce Foundation All rights reserved.

a. Draw a diagram of a rectangle with the same perimeter, but a larger area. D Write down the area of your rectangle. 0 61 X Ó 10 0 b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle. 0 6 2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches a. Is it possible to draw a rectangle with the same area as 3 inches the one on the right, but a larger perimeter? Explain your reasoning. 4 +60 8 inches Ó b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning. Deg Per an eter e t Student 7 5 inches a. Draw a diagram of a rectangle with the same perimeter, but a larger area. (_ Write down the area of your rectangle. incher 0 X 2 2 C nches Inche Ó 5 inches b. Draw a diagram of a rectangle with the same perimeter, but a smaller area. Write down the area of your rectangle. Thenes 5 X Z inci-2. The perimeter of this rectangle is 22 inches. The area of this rectangle is 24 square inches a. Is it possible to draw a rectangle with the same area as the one on the right, but a larger perimeter? 3 inches Explain your reasoning. because does TH 8 inches about not the leghth matter or height. X b. Is it possible to draw a rectangle with the same area, but a smaller perimeter? Explain your reasoning. Can perimeter because be am impe

Grade 6 – 2008 Copyright © 2008 by Noyce Foundation All rights reserved.

Can you figure out what the student is calculating in part 1. In 2b the student gives the perimeter for the correct rectangle, but no dimensions. What do you think the student understands?







Grade 6 – 2008 Copyright © 2008 by Noyce Foundation All rights reserved.

Ideas for Action Research 2 – Problem of the Month

One interesting task to help students stretch their thinking about 3-dimensional shapes is the problem of the month: **Surrounded and Covered**, *from the Noyce Website:*

www.noycefdn.org/math/members/POM/pom.html

Ask students to work individually or in teams to solve the problem. Have them make posters of calculations they made, their conclusions, and graphics or visuals to support their thinking. The poster might also include other ideas they want to explore or conjectures they haven't had time to test. The purpose is to give them some complex mathematical thinking, that requires persistence, willingness to make mistakes, edit and revise, and is worth understanding the thinking of others. Now give students a chance to investigate part D of this task.

Level D:

Tex has a home on the range (where the deer and antelope play). He wants to build a rectangular corral for his horses. IIe only has 170 feet of fencing. What size of corral should be built to make sure the horses have the most room? List the dimensions and area. Justify why you know you know the corral is as large as possible. Explain how you figured it out.

Tex gets a better idea. He has a barn on his ranch that is 80 feet by 120 feet. He decides to build the corral using one side of the barn and the 170 feet of fencing. What should be the dimensions of the corral now? What is the area? Explain your mathematical reasoning.

A year later, Tex needs a second corral. This time he has 240 feet of fencing. He doesn't want to use the other side of the barn, because it is near a small pond. He picks out a new location and realizes that he does not need to make the corral a rectangle. He designs a corral the shape of a hexagon. What are the lengths of the sides and what is the area of the corral? Explain how you found your answer.

Tex thinks that maybe another shape would make an even larger area for his corral. Determine what the shape should be and its area and dimensions. Justify your answer using mathematical reasoning.