Measurement STANDARD for Grades 9–12

Instructional programs from prekindergarten through grade 12 should enable all students to—

Expectations

In grades 9–12 all students should–

Understand measurable attributes of objects and the units, systems, and processes of measurement	• make decisions about units and scales that are appropriate for problem situations involving measurement.
Apply appropriate techniques, tools, and formulas to determine measurements	• analyze precision, accuracy, and approximate error in measurement situations; • understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders; • apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations; • use unit analysis to check measurement computations.

Measurement

Students should enter grades 9–12 with a good understanding of measurement concepts and well-developed measurement skills. In addition to reading measurements directly from instruments, students should have calculated distances indirectly and used derived measures, such as rates.

Opportunities to use and understand measurement arise naturally during high school in other areas of mathematics, in science, and in technical education. Measuring the number of revolutions per minute of an engine, vast distances in astronomy, or microscopic molecular distances extends students' facility with derived measures and indirect measurement. Calculator- and computer-based measurement instruments facilitate the collection, storage, and analysis of real-time measurement data. Through logarithmic scaling, students can graphically represent a relatively large range of measurements. Insight into formulas for the volume or surface area of a cone or a sphere can be gained by applying methods of successive approximation. These aspects of measurement, along with considerations of precision and error, are important in the students' high school experience.

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Understand measureable attributes of objects and the units, systems, and processes of measurement

Students should enter high school adept at using rates to express measurements of some attributes. Cars and buses travel at velocities expressed in miles per hour or kilometers per hour, the growth of plants is recorded in centimeters per day, and birth rates are often reported in births per 1000 people. By the time students reach high school, they should be ready to make sound decisions about how quantities should be measured and represented, depending on the situation and the problem under consideration. For example, the velocity of an insect measured as 4 centimeters a second is easier to understand than a velocity of 0.00004 kilometers a second, although they are equal. Students extend their understanding of measurement in the sciences, where many measurements are indirect. For example, they can determine the height of a bridge if they know it takes three seconds for a ball dropped off the bridge to hit the water below.

With the widespread use of calculator and computer technologies for gathering and displaying data, students should understand that selections of scale and viewing window become important choices. For example, the line $f(x) = x$ appears to intersect the two coordinate axes at an angle of 45 degrees only when the horizontal and vertical scales are the same. Likewise, circles viewed on screens where the horizontal and vertical scales differ look like ellipses. The local and global behavior of a function shown in different viewing windows can appear very different; at times, small differences in choices can lead to significant differences in the visual messages. Teachers can help students understand how to make strategic choices about scale and viewing window so that they can solve the problems they are addressing most effectively.

Nonlinear scales help represent some naturally occurring phenomena. For example, human ears have the ability to differentiate among sounds of low intensity that are very close together, but they do not distinguish as well among sounds of high intensity. Consequently, measures of sound intensity are often displayed on a logarithmic scale, tied to equally spaced decibel units as shown in the chart in figure 7.20 (units are in newtons per square meter). Students should be able to compare, for example, the loudness of a whisper (20 decibels) with that of a vacuum cleaner (80 decibels), noting that for each ten-decibel increase, the sound intensity increases by a factor of 10.

Fig. **7.20.**

A chart showing that the decibel scale is logarithmic

			Barely audible whisper ↓						Vacuuming ↓			Listening to Walkman ↓			Jet engines heard from nearby ↓
Decibels	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140
Sound intensity in newtons per m^2	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{3}	10^{4}	10^{5}	10^{6}	10^{7}	10^{8}	10^{9}

Apply appropriate techniques, tools, and formulas to determine measurements

High school students should be able to make reasonable estimates and sensible judgments about the precision and accuracy of the values they report. Teachers can help students understand that measurements of continuous quantities are always approximations. For example, suppose a situation calls for determining the mass of a bar of gold bullion in the shape of a rectangular prism whose length, width, and height are measured as 27.9 centimeters, 10.2 centimeters, and 6.4 centimeters, respectively. Knowing that the density is 19 300 kilograms per cubic meter, students might compute the mass as follows:

$$\begin{aligned}\text{Mass} &= (\text{density})\cdot(\text{volume})\\ &= \left(19300\,\frac{\text{kg}}{\text{m}^3}\right)\cdot\left(\frac{1}{10^6}\,\frac{\text{m}^3}{\text{cm}^3}\right)\cdot\left(1821.312\ \text{cm}^3\right)\\ &= 35.1513216\ \text{kg}\end{aligned}$$

The students need to understand that reporting the mass with this degree of precision would be misleading because it would suggest a degree of accuracy far greater than the actual accuracy of the measurement. Since the lengths of the edges are reported to the nearest tenth of a centimeter, the measurements are precise only to 0.05 centimeter. That is, the edges could actually have measures in the intervals 27.9 ± 0.05, 10.2 ± 0.05, and 6.4 ± 0.05. If students calculate the possible maximum and minimum mass, given these dimensions, they will see that at most one decimal place in accuracy is justified.

As suggested by the example above, units should be reported along with numerical values in measurement computations. The following problem requires both an understanding of derived measurements and facility in unit analysis—keeping track of units during computations:

> While driving through Canada in the late 1990s, a U.S. tourist put 60 liters of gas in his car. The gas cost Can$0.50 a liter (Can$ stands for Canadian dollars). The exchange rate at that time was Can$1.49 for each US$1.00 (United States dollar). The price for a gallon of gasoline in the

United States was US$0.99. The driver wanted to compare prices and decide whether a tank of gas was cheaper in the United States or Canada.

The cost of the gasoline in Canada follows:

$$\frac{\text{Can\$0.50}}{1\cancel{L}} \cdot 60\cancel{L} = \text{CAN\$30.00}$$

Teachers can help students recognize that in order to compute the cost of the same quantity of gasoline in the United States, it is necessary to convert between both monetary systems and units of volume. Thus, in addition to knowing the exchange rate, it is necessary to know that there are approximately 3.79 liters in each gallon of gas. The cost of 60 liters of gasoline at the U.S. price can then be seen to be

$$\frac{\cancel{\text{US}}\$0.99}{1\,\cancel{\text{gal}}} \cdot \frac{1\,\cancel{\text{gal}}}{3.79\cancel{L}} \cdot 60\cancel{L} \cdot \frac{\text{Can\$1.49}}{\cancel{\text{US}}\$1.00} \approx \text{Can\$23.35},$$

so the gas is less expensive in the United States. This computation illustrates how unit analysis can be helpful in keeping track of the conversions.

An important measurement idea, which also helps to establish the groundwork for some fundamental ideas of calculus, is that the measurements of some quantities can be determined by sequences of increasingly accurate approximations. For example, suppose that students are exploring ways to find the volumes of three-dimensional solids. Students should know that the volume of any right cylinder is the product of its height and the area of its base. Thus the volume of a right circular cylinder would be $\pi r^2 h$, where r is the radius of its base and h is its height. But how could students determine the volume of a cone?

To illustrate, the right circular cone shown in figure 7.21 has base radius 5 centimeters and height 10 centimeters. Using similar triangles, students should be able to see that slicing the cone parallel to the base at 2-centimeter intervals yields four nearly cylindrical disks and a small cone, each of height 2 centimeters. Each of those five figures would fit completely inside a cylinder that is 2 centimeters high and that has the same radius as its bottom cross section. Hence, the cumulative volume is less than $2(\pi \times 5^2 + \pi \times 4^2 + \pi \times 3^2 + \pi \times 2^2 + \pi \times 1^2)\ \text{cm}^3 = 110\pi\ \text{cm}^3$. At the same time, each of the five figures contains as a subset a cylinder that has the radius of its top cross section. Thus the cumulative volume must be at least $2(\pi \times 4^2 + \pi \times 3^2 + \pi \times 2^2 + \pi \times 1^2 + \pi \times 0^2)\ \text{cm}^3 = 60\pi\ \text{cm}^3$. Repeating the process with 1-centimeter-thick slices would help students see that as they take thinner slices of the cone, the overestimates and underestimates of the volume get increasingly close to each other. (Indeed, the averages of the underestimates and overestimates rapidly approach the actual volume of $(83\ 1/3)\pi\ \text{cm}^3$.) Informally, such experiences serve as an introduction to the idea of approximation by using upper and lower bounds and to the idea of limits. Whether or not these ideas are pursued later in formal coursework, they can introduce powerful ways of thinking about mathematical phenomena and help students establish a basic familiarity with core ideas of calculus.

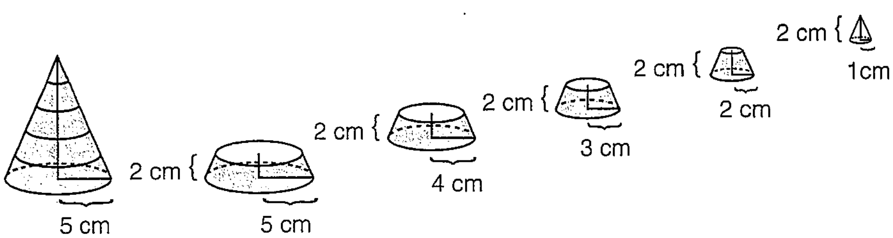

Fig. **7.21.**

Slices of a cone can be used to approximate the volume of the cone by using upper and lower bounds.