Measurement Standard for Grades 6–8

Instructional programs from prekindergarten through grade 12 should enable all students to—

Expectations

In grades 6–8 all students should–

Understand measurable attributes of objects and the units, systems, and processes of measurement	• understand both metric and customary systems of measurement; • understand relationships among units and convert from one unit to another within the same system; • understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.
Apply appropriate techniques, tools, and formulas to determine measurements	• use common benchmarks to select appropriate methods for estimating measurements; • select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision; • develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes; • develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders; • solve problems involving scale factors, using ratio and proportion; • solve simple problems involving rates and derived measurements for such attributes as velocity and density.

Measurement

Students bring to the middle grades many years of diverse experiences with measurement from prior classroom instruction and from using measurement in their everyday lives. In the middle grades, students should build on their formal and informal experiences with measurable attributes like length, area, and volume; with units of measurement; and with systems of measurement.

Important aspects of measurement in the middle grades include choosing and using compatible units for the attributes being measured, estimating measurements, selecting appropriate units and scales on the basis of the precision desired, and solving problems involving the perimeter and area of two-dimensional shapes and the surface area and volume of three-dimensional objects. Students should also become proficient at measuring angles and using ratio and proportion to solve problems involving scaling, similarity, and derived measures.

Measurement concepts and skills can be developed and used throughout the school year rather than treated exclusively as a separate unit of study. Many measurement topics are closely related to what students learn in geometry. In particular, the Measurement and Geometry Standards span several important middle-grades topics, such as similarity, perimeter, area, volume, and classifications of shape that depend on side lengths or angle measures. Measurement is also tied to ideas and skills in number, algebra, and data analysis in such topics as the metric system of measurement, distance-velocity-time relationships, and data collected by direct or indirect measurement. Finally, many measurement concepts and skills can be both learned and applied in students' study of science in the middle grades.

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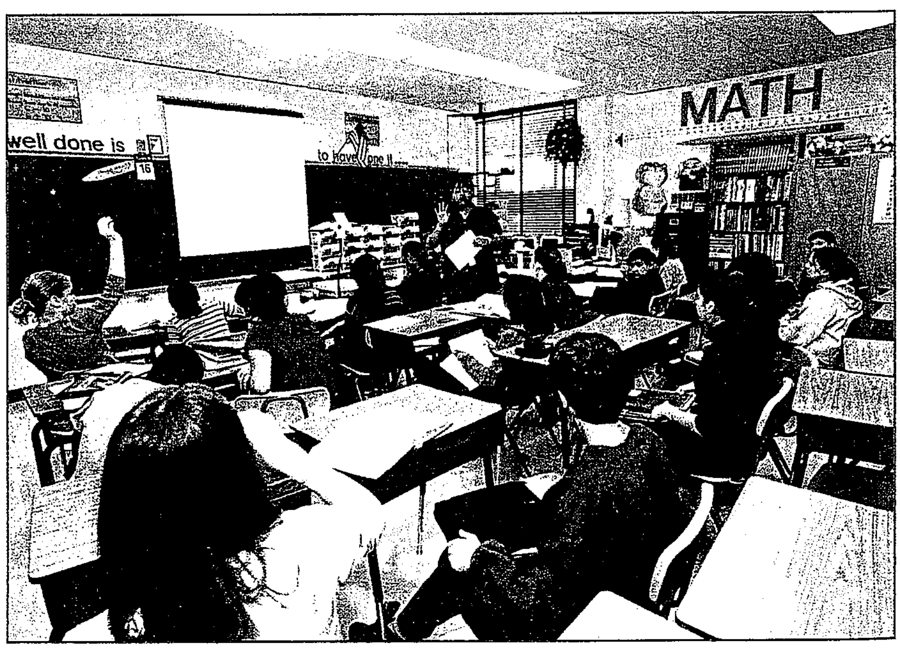
Understand measurable attributes of objects and the units, systems, and processes of measurement

From earlier instruction in school and life experience outside school, middle-grades students know that measurement is a process that assigns numerical values to spatial and physical attributes such as length. Students have some familiarity with metric and customary units, especially for length. For example, they should know some common equivalences within these systems, such as 100 centimeters equals 1 meter and 36 inches equals 3 feet, which equals 1 yard. In the middle grades they should become proficient in converting measurements to different units within a system, recognizing new equivalences, such as 1 square yard equals 9 square feet and 1 cubic meter equals 1 000 000 cubic centimeters. Work in the metric system ties nicely to students' emerging understanding of, and proficiency in, decimal computation and the use of scientific notation to express large numbers. When moving between the metric and customary systems, students are likely to find approximate equivalents—a quart is a little less than a liter and a yard is a little less than a meter—both useful and memorable.

Students in grades 6–8 should become proficient in selecting the appropriate size and type of unit for a given measurement situation. They should know that it makes sense to use liters rather than milliliters when determining the amount of refreshments for the school dance but that milliliters may be quite appropriate when measuring a small amount of a liquid for a science experiment.

In the middle grades, students expand their experiences with measurement. Although students may have developed an initial understanding of area and volume, many will need additional experiences in measuring directly to deepen their understanding of the area of two-dimensional shapes and the surface area and volume of three-dimensional objects. Even in the middle grades, some measurement of area and volume by actually covering shapes and filling objects can be worthwhile for many students. Through such experiences, teachers can help students clarify concepts associated with these topics. For example, many students experience some confusion about why square units are always used to measure area and cubic units to measure volume, especially when the shapes or objects being measured are not squares or cubes. If they move rapidly to using formulas without an adequate conceptual foundation in area and volume, many students could have underlying confusions such as these that would interfere with their working meaningfully with measurements.

Frequent experiences in measuring surface area and volume can also help students develop sound understandings of the relationships among attributes and of the units appropriate for measuring them. For example, some students may hold the misconception that if the volume of a three-dimensional shape is known, then its surface area can be determined. This misunderstanding appears to come from an incorrect overgeneralization of the very special relationship that exists for a cube: If the volume of a cube is known, then its surface area can be uniquely determined. For example, if the volume of a cube is 64 cubic units, then its surface area is 96 square units. But this relationship is not true for rectangular prisms or for other three-dimensional objects in general. To address and correct this misunderstanding, a teacher can have students use a fixed number of stacking cubes to build different rectangular prisms and then record the corresponding surface area of each arrangement. Because the number of cubes is the same, the volume is identical for all, but the surface area varies. Although a single counterexample is



sufficient to demonstrate mathematically that volume does not determine surface area, one example may not dispel the misconception for all students. Some students will benefit from repeating this activity with several different fixed volumes. Students can reap an additional benefit from this activity by considering how the shapes of rectangular prisms with a fixed volume are related to their surface areas. By observing patterns in the tables they construct for different fixed volumes, students can note that prisms of a given volume that are cubelike (i.e., whose linear dimensions are nearly equal) tend to have less surface area than those that are less cubelike. Experiences such as this contribute to a general understanding of the relationship between shape and size, extend students' earlier work in patterns of variation in the perimeters and areas of rectangles, and lay a foundation for a further examination of surface area and volume in calculus.

Students who have had experience in determining and using benchmark angles are less likely to misread a protractor.

Apply appropriate techniques, tools, and formulas to determine measurements

When students measure an object, the result should make sense; estimates and benchmarks can help students recognize when a measurement is reasonable. Students can use their sense of the size of common units to estimate measurements; for example, the height of the classroom door is about two meters, it takes about ten minutes to walk from the middle school to the high school, or the textbook weighs about two pounds. They should also be able to use commonly understood benchmarks to estimate large measurements; for instance, the distance between the middle school and the high school is about the length of ten football fields.

Students should become proficient in composing and decomposing two- and three-dimensional shapes in order to find the lengths, areas, and volumes of various complex objects. In addition, they should develop an understanding of different angle relationships and be proficient in measuring angles. Toward this end, they should learn to use a protractor to measure angles directly. Just as lower-grades students need help learning to use a ruler to measure length, so middle-grades students also need help with the mechanics of using a protractor—aligning it properly with the vertex and sides of the angle to be measured and reading the correct size of the angle on the scale. Students who have had experience in determining and using benchmark angles are less likely to misread a protractor. Estimating that an angle is less than 90 degrees should prevent a student from misreading a measurement of 150 degrees for a 30-degree angle. Students can develop a repertoire of benchmark angles, including right angles, straight angles, and 45-degree angles. They should be able to offer reasonable estimates for the measurement of any angle between 0 degrees and 180 degrees. Checking the reasonableness of a measurement should be a part of the process.

In the middle grades, students should also develop an understanding of precision and measurement error. By examining and discussing how objects are measured and how the results are expressed, teachers can help their students understand that a measurement is precise only to one-half of the smallest unit used in the measurement. That is, when students say that the length of a book, to the nearest quarter inch, is 12 1/4 inches, they should be aware that the measurement could be off by 1/8 inch. Thus, the absolute error in the measurement is ±1/8 inch in this instance. Similarly, if they use a protractor to measure angles to the nearest degree, they will be precise within 1/2 degree.

An understanding of the concepts of perimeter, area, and volume is initiated in lower grades and extended and deepened in grades 6–8. Whenever possible, students should develop formulas and procedures meaningfully through investigation rather than memorize them. Even formulas that are difficult to justify rigorously in the middle grades, such as that for the area of a circle, should be treated in ways that help students develop an intuitive sense of their reasonableness.

One particularly accessible and rich domain for such investigation is areas of parallelograms, triangles, and trapezoids. Students can develop formulas for these shapes using what they have previously learned about how to find the area of a rectangle, along with an understanding that decomposing a shape and rearranging its component parts without overlapping does not affect the area of the shape. For example, figure 6.23 illustrates how students could use their knowledge of the area of a rectangle to generate formulas for the area of a parallelogram and a triangle. Once students develop these formulas, they can also generate a formula for the area of a trapezoid. A teacher might have students begin working with isosceles trapezoids and then try to generalize their formula for any trapezoid. As suggested in figure 6.24, students can use what they know about rectangles, parallelograms, and triangles in several different ways. They might decompose an isosceles trapezoid into two triangles and a rectangle and rearrange these shapes to form a rectangle, or they might duplicate the trapezoid and arrange the two shapes to form a parallelogram.

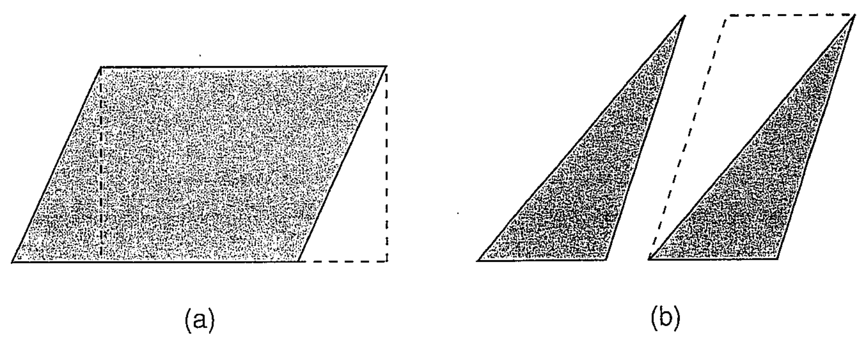


Fig. **6.23.**

Students can use their knowledge of the area of a rectangle to generate a formula for the area of a parallelogram (a) and for the area of a triangle (b).

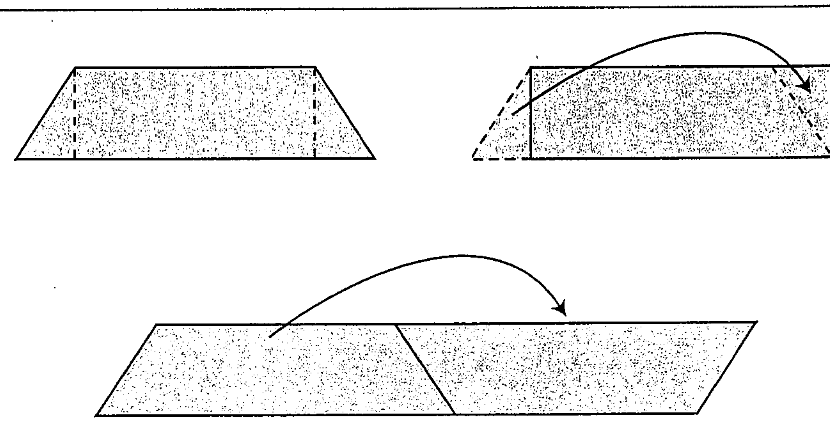


Fig. **6.24.**

An isosceles trapezoid can be decomposed and rearranged or duplicated in order to find a formula for its area.

Students in the middle grades should also develop formulas for the surface areas and volumes of three-dimensional objects. Teachers can help students develop formulas for the volumes of prisms, pyramids, and cylinders and for the surface areas of right prisms and cylinders by having them construct models, measure the dimensions, estimate the areas and volumes, and look for patterns and relationships related to lengths, areas, and volumes. In their work with three-dimensional objects, students can make use of what they know about two-dimensional shapes. For example, they can relate the surface area of a three-dimensional object to the area of its two-dimensional net. Students might determine the surface area of a cylinder by determining the area of its net (the two-dimensional figure produced by cutting the cylinder and laying it flat), which consists of a rectangle and two circles (see fig. 6.25). Thus, students can develop a formula for the surface area of a cylinder because they can find the areas of the circles and the rectangle that compose its net.

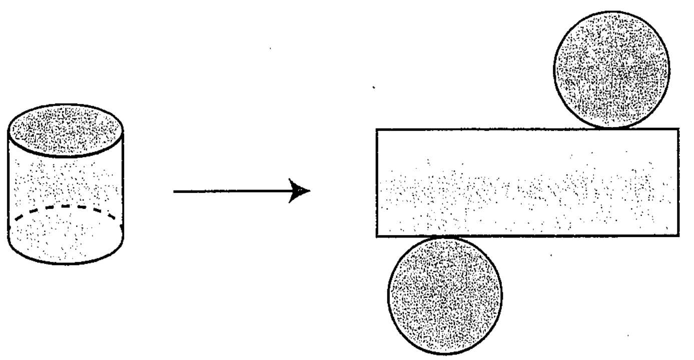


Fig. **6.25.**

Students can determine the surface area of a cylinder by determining the area of its net.

It is important that middle-grades students understand similarity, which is closely related to their more general understanding of proportionality and to the idea of correspondence. Students can use measurement to explore the meaning of similarity and later to apply the concept to solve problems. The important observation that the measurements of the corresponding angles of similar shapes are equal is often a starting point for work with similarity. Measurement is also useful for determining the relationships between the side lengths and the perimeters and areas of similar shapes and the surface areas and volumes of similar objects. Students need to understand that the perimeters of pairs of similar shapes are proportional to their corresponding side lengths but that their areas are proportional to the squares of the corresponding side lengths. Similarly, through investigation they should recognize that the surface areas of similar objects are proportional to the squares of the lengths of their corresponding sides but that their volumes are proportional to the cubes of those lengths.

Problems that involve constructing or interpreting scale drawings offer students opportunities to use and increase their knowledge of similarity, ratio, and proportionality. Such problems can be created from many sources, such as maps, blueprints, science, and even literature. For example, in *Gulliver's Travels*, a novel by Jonathan Swift, many passages suggest problems related to scaling, similarity, and proportionality. Another interesting springboard for such problems is "One Inch Tall," a poem by Shel Silverstein (1974) (see fig. 6.26).

Fig. **6.26**

"One Inch Tall," by Shel Silverstein (1974). Used with permission.

One Inch Tall

If you were only one inch tall, you'd ride a worm to school.
The teardrop of a crying ant would be your swimming pool.
A crumb of cake would be a feast
And last you seven days at least,
A flea would be a frightening beast
If you were one inch tall.

If you were only one inch tall, you'd walk beneath the door,
And it would take about a month to get down to the store.
A bit of fluff would be your bed,
You'd swing upon a spider's thread,
And wear a thimble on your head
If you were one inch tall.

You'd surf across the kitchen sink upon a stick of gum.
You couldn't hug your mama, you'd just have to hug her thumb.
You'd run from people's feet in fright,
To move a pen would take all night,
(This poem took fourteen years to write—
'Cause I'm just one inch tall).

—*Shel Silverstein*

In connection with the poem, a teacher could pose a problem like the following:

> Use ratios and proportions to help you decide whether the statements in Shel Silverstein's poem are plausible. Imagine that you are the person described in the poem, and assume that all your body parts changed in proportion to the change in your height. Choose one of the following to investigate and write a complete report of your investigation, including details of any measurements you made or calculations you performed:
>
> - In the poem the author says that you could ride a worm to school. Is this statement plausible? Would it be true that you could ride a worm if you were 1 inch tall? Use the fact that common earthworms are about 5 inches long with diameters of about 1/4 inch.
> - In the poem the author says that you could wear a thimble on your head. Would this be true if you were only 1 inch tall? Use one of the thimbles in the activity box to help you decide.

To solve this problem, students need to use proportionality to imagine a scale model of a student shrunk to a height of 1 inch. They need to consider the resulting circumference of a student's head or the resulting length of a student's legs in relation to the diameter of a cross section of a worm. Because middle-grades students' heights vary greatly, scale factors will vary greatly within a class, which can generate a lively discussion. A student who is 4 feet 8 inches tall would use a 56:1 ratio as a scale factor; in contrast, a student who is 5 feet tall would use a scale factor based on a 60:1 ratio. After discussing this problem and pointing out that an author can legitimately use poetic license to create images that do not conform to reality, a teacher might extend the investigation by asking students to evaluate the plausibility of other statements in the poem that intrigue them. Alternatively, a teacher might select a statement, for example, "And it would take about a month to get down to the store," which refers to a rate given as a distance-time relationship. Students could use a stopwatch and a tape measure to get distance-time readings for a typical student. They could then determine how far away the store would need to be in order for the assertion to be plausible, given a proportional change in the rate of walking for a student shrunk to 1 inch.

Students should also have opportunities to consider other kinds of rates, such as monetary exchange rates, which can afford practice with decimal computation and experience with ratios and rates expressed as single numbers. Experience with cost-per-item rates are also valuable; see the examples in the "Problem Solving" and "Algebra" sections in this chapter.

Teachers can use technological tools such as computer-based laboratories (CBLs) to expand the set of measurement experiences, especially those involving rates and derived measures, and to relate measurement to other topics in the curriculum. For example, using the CBL to measure a student's distance from an object as she walks away from or toward it and plotting the corresponding points on a distance-time graph can be very instructive. Different paths generate different graphs. Different start-end points and variations in speed can also affect the graphs. Students could generate many such graphs with specific kinds of variation and then discuss the graphs to help them relate this experience to their developing understandings of linear relationships, proportionality, and slopes and rates of change. Questions such as the following might be useful:

- For which graphs does the relationship between distance and time appear to be linear? For which is the relationship nonlinear? Why?
- For the graphs that depict a linear relationship, how does the speed at which the person walks appear to affect the graph? Why?
- Which graphs portray a proportional relationship between distance and time? Are there any graphs that depict proportional relationships that are not also linear? Are there any that depict linear relationships that are not also proportional? Why?
- Would it be possible to generate a distance-time graph that depicts a relationship that is linear but not proportional? That is proportional but not linear? Why?

A teacher can use such experiences, whether in the mathematics class or in collaboration with a science teacher, not only to enrich students' understanding of topics in measurement but also to provide a springboard for the study of data representation and analysis.