From Classroom Discussions to Group Discourse

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The vision to transform mathematics classrooms into learning communities in which students engage in mathematical discourse is a remarkable hallmark of the current movement, led by the National Council of Teachers of Mathematics, to reform mathematics education (NCTM 1991, 2000). According to NCTM, "the discourse of a classroom—the ways of representing, thinking, talking, agreeing and disagreeing—is central to what students learn about mathematics as a domain of human inquiry with characteristic ways of knowing" (NCTM 1991, p. 34). Indeed, both the *Principles and Standards for School Mathematics* (2000) and *Professional Standards for Teaching Mathematics* (1991) recommend that teachers of mathematics provide opportunities for children of all ages to participate in mathematical discourse.

One popular interpretation of this recommendation has been that teachers allow students to share their ideas in class in order to increase their interest and participation in learning (Lampert and Blunk 1998). Although this interpretation is not inaccurate, it provides a narrow view of what the *Standards* mean by the concept of "classroom discourse" and what they envision it can accomplish. Explicit in both sets of NCTM *Standards* is the link between *discourse* and a *learning community,* that is, a classroom environment that embodies a culture of learning in which everyone is involved in a collective effort of understanding (Bielaczyc and Collins 1999; Mercer, Wegerif, and Dawes 1999). Much emphasis is placed on the value of classroom discourse because of its perceived power in fostering a learning community. Conversely, the creation of a learning community is deemed important because of its potential for generating productive mathematical dialogues among learners. This crucial link, although frequently overlooked, provides a substantially richer context for understanding not only why students should be encouraged to share their ideas but also how their ideas should be treated and used in class.

In this article, we will argue that the discourse of a learning community as advocated by the *Standards* differs significantly not only in structure but also in content, purpose, and product from the type of dialogues commonly produced in mathematics classrooms. We will use vignettes from two high school geometry classrooms to illustrate these differences and to discuss multiple ways in which these differences affect the act of teaching and the quality of students' mathematical work.

CHARACTERIZING DISCOURSE OF A LEARNING COMMUNITY

Discourse is commonly thought of as a simple conversational exchange of ideas or relaxed discussions among individuals. Unlike casual conversation, however, discourse requires a combination of both reflection and action. That is, during the exchange of ideas, participants attempt both to gain insight into the conceptions of others and to influence them (NCTM 1991). Burbules (1993), pointing at this difference, characterized discourse as a communicative relationship between equals that requires *participation, commitment,* and *reciprocity. Participation* means there are opportunities in the dialogue for the individuals to become engaged, question others, try out new ideas, and hear diverse points of view. *Commitment* implies that the participants will be open to hearing the positions of other speakers. *Reciprocity* means a willingness to engage in an equilateral exchange with others. In this mode, the *structure* of discourse is multidirectional and responsive. The *content* of the dialogues is dynamic, connected, and unscripted. The *purpose* of the dialogue is to participate and engage others in deep inquiry into the meaning of things. This posture leads to significant changes in how individuals view the topic under consideration and their relationships with it.

Rich dialogue occurs when participants are ready to consider the possibility of different interpretations and meanings, when they analyze how their own ideas differ from those of their conversational partners, and when they allow themselves to consider the ways in which their prior understandings might be distorted. Thus, a major product of discourse is the transformation of the participants. The *Standards* promote and support this type of transformational discourse. Not all classroom dialogues share these features of discourse or lead to the same outcomes.

STRUCTURE, CONTENT, PURPOSE, AND PRODUCT OF DISCOURSE IN DIFFERENT CLASSROOMS

Structure of Discourse

In a majority of mathematics classrooms, discussions frequently follow a traditional format: The teacher initiates a dialogue by posing a question, and students either volunteer answers or are called on by the teacher. The teacher reacts to the students' ideas by providing corrective feedback. Ultimately, the teacher shares or approves of a method that she or he considers correct. The students take note of these answers and are expected to reference them in future contexts. This type of classroom discussion is not reciprocal, since the teacher determines the quality and quantity of student participation and contributions. In fact, the teacher decides which student ideas should be pursued or abandoned. In contrast, in a learning community, student answers are used to extend instruction (NCTM 1991). The teacher allows students to participate freely in dialogues and to make decisions about adequacy and efficiency of ideas that their peers offer. The discourse of this class is multidirectional and responsive. The teacher's contribution to classroom discourse is to intensify the mathematical substance of the group discussions rather than to reduce the cognitive load of tasks on which students work.

Content of Discourse

In traditional classroom settings, the content of discourse is predictable and polished. It is focused on helping students arrive at answers already approved by authorities, such as the teacher and the textbook. In addition, the pressure that comes from following timelines for curriculum coverage and keeping pace with the order in which mathematical topics are introduced in textbooks can impinge upon the teacher's time and thus compromise the quality of students' mathematical work. In the course of one single class session, for example, students may tackle multiple problems, but they may not be asked to examine connections between these problems and their solutions or even between the current problems and previous work. Even when these connections are discussed, they are frequently shared with students through carefully designed teacher presentations with little authentic contribution by learners. In contrast, discourse within a learning community focuses on seeking connections among ideas and distinguishing valid from invalid arguments (NCTM 1991). Individuals come together with mutual understanding and shared interest to construct knowledge through dialogues that are both intellectually challenging and time intensive. The content of student dialogue is shaped by the knowledge they gain from their interactions. Therefore, the direction or destination of student discourse is not always predictable. The teacher builds on student ideas to design curriculum and instruction that advance their understanding of mathematics, rather than superimposing a predetermined structure on their thinking.

Purpose of Discourse

The primary purpose of discourse in a traditional classroom is to transfer information. The teacher depends on dialogues with students to monitor their progress. She relies on student input (or lack thereof) to decide on her next teaching move: whether to review certain topics, provide additional examples, give additional assignments, or proceed with covering the next topic. In turn, students listen to the teacher and record the information she provides, trusting that the shared information is valuable. These outcomes of classroom dialogues certainly are important; but the ultimate goal of this type of discourse is to standardize students' thinking. In contrast, the purpose of discourse within a learning community is to assist both the teacher and the students in learning more about the subject. Student conversation is focused on exploring and explaining mathematical ideas and their connections. The discourse of the class is a sincere effort to establish new knowledge, seek new understanding, and inquire (NCTM 1991). Assessing student understanding in light of their discourse is only one of many purposes that classroom dialogues serve.

Product of Discourse

The primary outcome of discourse in traditional mathematics classrooms is the dissemination of facts about the discipline and those mathematical techniques that either the teacher or the textbook characterizes as efficient and elegant. The teacher carefully designs and delivers lectures to ensure that mathematical truths are clearly communicated with students. This form of knowledge sharing is not transformational, since it may not lead to substantial change in the way students or teachers think about mathematics. In contrast, discourse of a learning community involves students in dialogues in order to construct, negotiate, and verify mathematical ideas (Cobb and Bauersfeld 1995). The product of discourse is the development of shared understandings, new insights, and a deeper analysis of mathematics on the part of both the teacher and the student (Lampert and Blunk 1998). While the teacher is a participant in discourse, he or she is also the most knowledgeable mathematician in the group and simultaneously carries the responsibility to share needed conventional knowledge of the discipline with students and to help them synthesize ideas. Through careful and intentional interventions, she ensures that students indeed reach these shared understandings and insights.

TWO EXAMPLES: THE MEDIAN PROBLEM

Let us illustrate the differences noted above by considering an example of each type of discussion in two mathematics classrooms. The teaching episodes we describe in this section come from different geometry classrooms. The teachers, Tom and Lyle, taught in the same school and the student populations were consistent across both sections. Although both Tom and Lyle regularly encouraged students to share their ideas in class, Lyle had tried from the start of the academic year to create a learning community in which collaborative discourse guided the unfolding of mathematics as well as student learning in class. Tom, however, did not share this particular instructional goal. He used class discussions as a means to engage students in his lessons and to monitor their progress.

The following problem was the focus of work in both classrooms:

> Is this statement true or false? "In an isosceles triangle a median divides the triangle into two regions of equal area."

Students had been given the problem as a homework assignment and had been asked to come to class prepared to share their answers. In both classes, the teachers initiated the classroom dialogues, asking for student volunteers to share solutions. Discussion in both classes began as the first student volunteer in each section presented an identical solution to the problem. The subsequent mathematical interactions of the group and the outcomes of discussions were influenced by how this initial solution was treated and used by the teacher to motivate student learning.

Section 1: Tom's class

Tom reads the problem to the group and asks if students had worked on the problem. None of the students volunteers to show his or her work, so Tom encourages them to participate.

Tom: I don't want to just show you how to solve this problem. We need a volunteer. It doesn't matter if you have not solved it completely.

*One of the students, Kevin, volunteers to share his work. He instructs Tom to draw a triangle on the board and to label its vertices. Tom asks Kevin to come to the board to present his work. Kevin produces a drawing and explains it [see **fig. 1a**].*

Kevin: I said when we fold the triangle about the median AM_1, the areas of these two regions (*points at triangles ABM_1 and ACM_1*) match, so they are equal.

Tom looks at the group; but since no one reacts to Kevin's approach, he addresses Kevin.

Tom: What did you assume about the triangle? The problem says "isosceles triangles." Which of the sides are equal?
Kevin: Sides *AB, AC,* and *BC.*
Tom: So, what type of triangle is it? If all three sides are equal, what kind of triangle is it?

A student shouts out that it is equilateral.

Tom (*addressing the group*): You need to remember that not all isosceles triangles are equilateral. The problem said "isosceles triangle."
Evan: So your method does not work? Your method only works when it is equilateral?
Kevin (*responding to Evan*): If I changed it to just sides *AB* and *AC* equal then we can still fold it and they match.

Several students nod their heads in agreement. One student holds up a piece of paper on which she had drawn an isosceles triangle folded along the median AM_1 to illustrate Kevin's reasoning.

Tom: This folding method shows the two triangles are equal only for this case (*pointing at the median AM_1*), but it doesn't work when we consider another median, say BM_2, because in this case the triangles don't match. Do you see my point? Evan, did I answer your question?
Evan: Yes, I see why he is wrong.
Solmaz: I drew all three medians and tried to see if the triangles, the six triangles inside, ended up being equal, but I did not finish.
Tom: Okay, does anyone else have a solution?
Nassim: I don't know how we can find the areas without knowing the size of the triangle.
Tom: Do we need specific measurements to verify this statement?

Nassim does not respond.

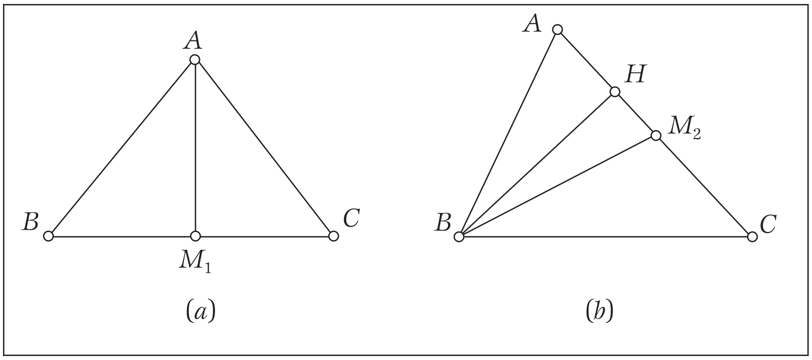

Fig. 1 Initial models used in Tom's class

Sara: I said when we draw a median we create two triangles in the original triangle. The bases of these two triangles are equal because of the property of midpoint, and then they share the same altitude, so if we use area = 1/2(height • base), then we get the same areas.

Tom draws a triangle to illustrate Sara's argument.

Tom: Sara says if we use the definition of the median and consider the common altitude of triangles ABM_2 and BCM_2, then we have equal areas [*see **fig. 1b***]. This is really slick: a generalized argument that applies to any of the medians in the triangle, right? Do you see it?

A student jokingly refers to Sara as the "brains" and asks how she came up with the idea. Sara explains that the hardest thing for her was noticing that triangles ABM_2 and BCM_2 shared the same altitude. Tom asks students to finish recording Sara's proof. Students are then advised to move on to the next homework problem.

Section 2: Lyle's class

Lyle reads the problem to the group and asks for students to share their solutions. Since none of the students volunteers to show his or her work, Lyle reads the problem aloud again. He asks how many people had worked on the problem. With the exception of two students, everyone raises a hand.

Lyle: So almost everyone worked on the problem, but no one is volunteering!
Naomi: I don't think my answer is what you want, like proving—like real proving.

She illustrates the same folding approach that Kevin had shown in Tom's class.

Lyle (*addressing the class*): What do you think? Is this demonstration enough to believe that any of the medians in this triangle divides the triangle into two regions of equal area?

Samantha: Are you saying sides *AB* and *AC* are equal?
Naomi (*elaborating on her argument*): Yes, I said it was okay for this median (*pointing at* AM_1), the one that cuts the unequal side, but it is not true for the other ones. Like when I drew BM_2, the two triangles did not match [*see* ***fig. 2***].
Meri: But the problem did not say that they had to be congruent; it said that their areas had to be equal. Two triangles can have different measurements but still have the same area.
Josh: But if *ABC* is an equilateral triangle then all the triangles are congruent.
Meri (*responding to Josh*): Yes, but this problem did not say that they HAD to be congruent! It said that they had to have the same area!

Everyone is silent for almost a minute. Lyle addresses the group again.

Lyle: These are both excellent comments. Josh says in an equilateral triangle, each median divides the triangle into two congruent triangles. Meri says that in our original statement it does not matter if the triangles are congruent—all we need to check is if their areas are equal. What do you think?
Samantha: I think they are both right. (*Several other students nod their heads in agreement*). I was talking to Noah about this. He can explain it better.
Will: I know what I did wrong. I thought they both had to be congruent, so I said the statement was wrong; but now that I listened to Meri, I think she is right. Like, we can have a triangle with height and base of 5 and 6, another one with height and base of 10 and 3. They both have the same area but they are not equal.
Noah: I think Josh is right about all 6 triangles being equal because here is what I did: I looked at the triangle and one of its medians and saw that the two triangles inside have the same altitude.

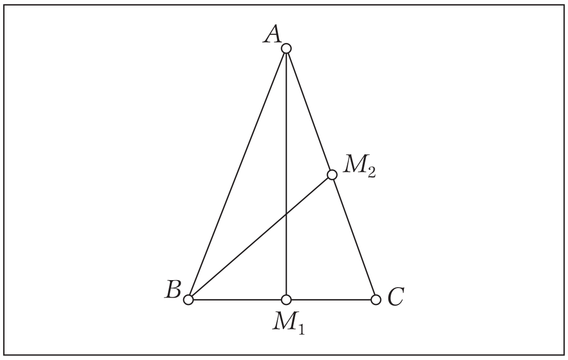

Fig. 2 Naomi's picture

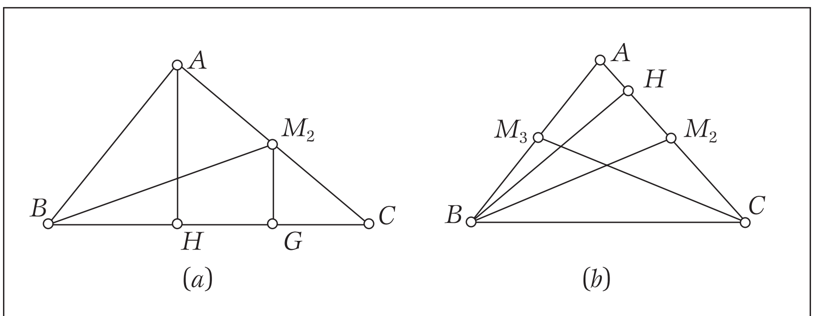

Fig. 3 Noah's model

Lyle draws the altitude AH, perpendicular to side BC as illustrated in ***figure 3a****. Noah corrects Lyle.*

Noah: No, those are not the ones I looked at (*goes to the board and produces* ***fig. 3b***). Then I said the areas of ABM_2 and CBM_2 are the same. See, $AM_2 = CM_2$ and *BH* is their common height. Here it really does not matter which median we consider, it always gives us two triangles of equal area. See, I could have looked at this other median (*draws* CM_3). Again, the altitude is the same for these two (*pointing at triangles* CAM_3 *and* CBM_3), their bases are equal, so their areas are equal [*see* ***fig. 3b***].
Hope: So what you are saying is that the statement is true for all triangles. It does not matter what kind of triangle we have, it does not matter if it is isosceles or not—because Noah's argument did not depend on the lengths of the sides of triangle *ABC.* So we could use the same method to show the areas would be equal in any triangle.
Noah: I guess you are right. I did not see that, but it is right.
Lyle: Good! Let's go back and see if there is a different way of looking at this problem. I am curious to know how we might justify Naomi's method.
Megan: Didn't we show that in an isosceles triangle the median is also the altitude? I mean the median that cuts the unequal side. So, with that we know triangles ABM_1 and ACM_1 are congruent by SSS congruency theorem.

Sounds of recognition arise from the group.

Jamie: When you drew those altitudes, *AH* and M_2G, I saw something. I thought, okay, the ratio of the altitudes is 2 to 1, because the ratio of *AC* to M_2C is 2 to 1. So, since the altitude of BM_2C is a half of *AH,* the area of triangle BM_2C is exactly a half of the area of *ABC,* so that leaves only a half for the area of triangle BM_2C. So, they must be equal.
Rosha: But Jamie, we don't know the ratio of the altitudes is 1/2. They may not be 1/2.
Jamie: I think it is 1/2 because M_2 is the midpoint of side *BC.*

Lyle steps away from the board and appears to be thinking about Jamie's method. Several students ask if

Lyle could give them a hint on how to prove Jamie's proposition.

Lyle: I never considered proving it this way

He draws a line from M_2 parallel to BC. Jamie and several others simultaneously shout "similar triangles," referencing their discussion of medial triangles from two weeks earlier. [Earlier in the term, students had considered the relationship between a triangle and its medial triangle, formed by connecting midpoints of its sides. At that time, the theorem stating that a segment connecting the midpoints of two sides of a triangle is parallel to the third side.]

Morgan: I think we can use this approach to show Naomi's folding method, because once we draw the medial triangle then we can show that areas of the triangles inside are equal. Like this. (*She produces **fig. 4** to illustrate her point.*)

Lyle advises students to get into small groups and review the different procedures that were presented in class. He instructs them to revisit each method and to decide, first, if each of the suggested arguments was complete and then, to refine each one if needed.

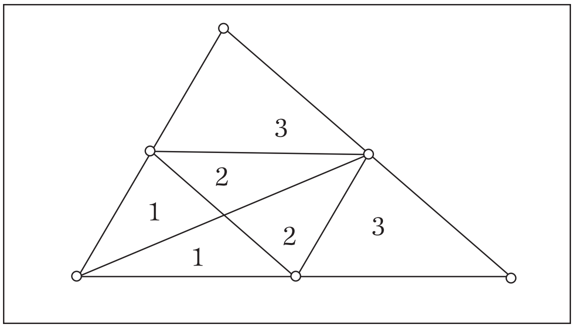

Fig. 4 Morgan's visual representation of Solmaz's conjecture

EXAMINING THE DISCOURSE OF THE TWO CLASSROOMS

Despite the presence of student conversation in both classrooms, there are substantial differences in the structure, content, and products of mathematical discussions of the two sessions.

Structure of Discourse

In the first vignette, following the initial invitation to share ideas, Tom explicitly controlled the classroom dialogues; he determined who participated in discussions, what information received attention, how ideas were verified, and which of the student contributions were used to assure that closure on the problem was achieved. The direction of dialogue was, for the most part, from Tom to particular students. The extent of commitment that students appeared to have made to discussions was to share their work (if directly asked), nod their heads in agreement, or to record the mathematical product that was either presented by Tom or approved by him (for instance, Sara's method). Even when Evan questioned the accuracy of Kevin's method, it was Tom who responded to Evan's question rather than allowing Kevin to defend his approach or elaborate on it. Once Tom approved Sara's proof, the discussion quickly ended. In his class the students were verbally engaged in discussions, but they were inherently an audience for the mathematics that Tom displayed.

In Lyle's class, peer discussion seemed more autonomous. After Lyle's initial invitation, students chose to participate in the dialogue and were responsive not only to Lyle's questions but to the mathematical ideas that their peers shared in class. The students addressed each other's work directly rather than filtering their comments through Lyle. Reciprocity to the shared ideas was evident as students changed their own solutions in light of peer comments. Lyle explicitly influenced the content and flow of discourse by repeating student ideas, noting the importance of particular comments, asking students to restate peers' propositions, and even choosing to remain silent when confronted with certain questions. His interventions promoted and supported student discussions.

Content of Discourse

In Tom's class, the content of discourse was channeled to direct students toward one answer. In fact, when Solmaz suggested her method for examining the areas of six smaller triangles, potentially a very productive approach, Tom did not pursue her idea. Thus, her method did not receive attention from her peers. Although Tom posed good questions, he did not appear to expect students to pursue them. The content of dialogue in Tom's class was not about finding connections among the various methods but on isolating those features that Tom considered valuable in arriving at the answer he considered correct. Students noted Tom's explanations and relied on him to determine the validity of their peers' ideas. Tom's responses restricted the content of student conversation, their exploration of the problem, and what they gained from ideas posed by different individuals.

In Lyle's class, the content of discourse was shaped by what students knew (their initial solutions) and what they learned in the course of their discussions. The classroom dialogue seemed to help students transform their understanding of the problem and lead to a deeper analysis of methods they could use to make sense of and solve the problem. Although Lyle intentionally posed questions to guide and structure students' work, the content of students' discourse was not limited to answering his questions. Notice also that students did not reach immediate closure on the problem. Their mathematical dialogue became increasingly more refined as they developed new insights that affirmed or refuted their previous ideas. Lyle provided students with the time and the intellectual space they needed to practice this process. Although he contributed to classroom dialogues, his interventions seemed to deepen students' inquiry rather than place closure on it.

Product of Discourse

Although Tom invited students to react to one another, he did not insist that they critically analyze each other's work or try to understand their peers' views. Students were not expected to challenge mathematical ideas or to confront one another's arguments. Therefore, the worth of the information that students shared was determined solely by Tom's judgment. The product of discussion in Tom's class was a correct method for solving one problem. In the process, Tom managed to remind students of some definitions. He shared what he considered to be the limitations of some of the methods that students offered. The class discussion does not allow us to deduce much about what students gained from the dialogues that took place in class or whether they reached a shared understanding of why some procedures were right or wrong.

Teachers need to help students develop the belief that they as individuals are responsible for understanding and sharing mathematics

In Lyle's class, student conversation focused on exploring and explaining mathematical ideas and their connections. Lyle participated in this discourse not only to monitor, assess, and guide students but also to develop new insights about the mathematics that his students presented and used. As students tried to make sense of each other's ideas, their own understanding of the problem seemed to evolve. As a result, their mathematical work became more sophisticated. Students not only shared their own solutions but also examined the validity of different methods their peers or the teacher offered. In the process, they influenced the development of mathematics in the classroom and the order in which it unfolded. Following the particular teaching episode we reported here, Lyle's students spent three additional class sessions discussing the various methods of their peers. In the course of their analysis, they posed and solved nearly fifteen different problems that addressed relationships among medians, altitudes, and angle bisectors of different triangles. These are significant mathematical outcomes.

NURTURING THE DEVELOPMENT OF A LEARNING COMMUNITY

Using student discussion in mathematics instruction is not a novelty in education. However, motivating productive discourse to advance the development of a learning community is certainly a new educational vision. Implementing this vision demands more than just allowing students to talk in class; it requires engaging students in authentic mathematical activities. Authentic mathematical activity involves students' adopting perspectives, beliefs, values, and expectations consistent with those of the mathematics community and using those to analyze and discuss problem situations. Students must become able to make and state mathematical observations on their own, take ownership of the thinking that must be done, and break away from the belief, fostered by much of the schooling process, that authority resides only in books and teachers. Teachers need to help students develop the belief that they as individuals are responsible for understanding and sharing mathematics. To do this, teachers can create contexts that allow for development of these perspectives and beliefs by giving students more responsibility for the mathematical context on which classroom instruction is based (McClain and Cobb 2001). By providing students the opportunity to work on open-ended problems instead of having them simply search for a calculation to find an answer, teachers can help students become more autonomous in their use of mathematics. Moreover, by insisting that peers work together to try to figure things out, teachers can influence students' perceptions about their role in the classroom and their expectations of peers.

Students do not automatically begin talking about mathematics in meaningful ways simply because they are presented with good tasks and asked to work and talk with each other (Lampert 2001). Teachers also need to help students learn how to talk with one another about mathematics in ways that are coherent and respectful by pointing out features of classroom conversations that are representative of the type of discourse they desire and by modeling for students those social and mathematical behaviors, including the norms of polite interaction, that are crucial to productive functioning of a learning community. To increase student sensitivity to what is being shared in class, teachers can emphasize the need to listen to ideas and ask questions about peers' propositions.

A critical consideration for all teachers is the involvement of and learning by all students in class, including those less mathematically advanced than others. In a learning environment that depends heavily on group discourse, teachers need to moderate group interactions to ensure that all students will have the opportunity to participate in and benefit from discussions. By asking questions that encourage students to reflect on mathematics that might have been implicit in their peers' discussions and by requiring students to verbalize what others have said and what they might have meant, teachers can make it easier for reluctant students to contribute to group discussions. Moreover, by restating and elaborating on student comments, teachers can pace the tempo of instruction to ensure that all students have the time they need to understand and digest proposed ideas. These actions also help to socialize students; they gain practice listening to, thinking about, and analyzing the comments of their peers. Once students become aware that others are interpreting and assigning significance to what they are saying, they will show greater precision in expressing themselves and interest in sharing their ideas (Rittenhouse 1998).

FINAL COMMENTS

Creating a learning community that supports and encourages students' authentic engagement in the construction of mathematical knowledge depends primarily upon the teacher's own efforts and instructional behaviors. Certainly, learners' perception of what the teacher values can determine the extent to which they participate in and benefit from discussions. If one hopes for students to develop the mathematical and social dispositions to act as a community of learners, then teachers must both support and model those ways of thinking and acting.

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